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Second Loop Corrections from Superheavy Gauge Sector to Gauge Coupling Unification

Dariusz Grech

Institute of Theoretical Physics
University of Wrocław, Maks Born Sq.9,
PL-50-204 Wrocław, POLAND

Abstract

We deal with extensions of the Standard Model (SM) adding horizontal interactions between particle generations. We calculate two loop corrections caused by the presence of coupling between hypothetical horizontal gauge bosons and matter field at high energy. It is shown that coupling of such extra bosons does not affect up to two loop level the positive features of unified and extended SM with horizontal symmetry discussed in former publications. Corrections from bosonic horizontal sector make about tenth part of those caused by fermionic sector. Although small they are however larger than accuracy of some electroweak measurements and therefore they might be important for future verification of various proposed horizontal models.

1 Introduction

Standard model of strong and electroweak interactions (SM) based on $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group has got many unquestionable achievments. Nevertheless it does not seem to be complete. Everyone agrees it contains too many arbitrary parameters. Their origin can in general be explained only in larger schemes which incorporate additional global or local symmetries. Such models beyond the SM can be divided into SUSY models - with Minimal Supersymmetric Standard Model (MSSM) as the best candidate, or schemes with extended gauge sector. In the latter type of theories the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group above the electroweak mass scale M_Z is extended to $SU(3)_C \times SU(2)_L \times U(1)_Y \times G$ group product where G is new exact gauge symmetry broken at some mass scale $M_G > M_Z$. Most popular left-right symmetric models and horizontal symmetry models belong to such class of theories. One should note that junctions between these models and SUSY can also be considered. How far will fermions and bosons associated with G change or explain parameters of SM?

We will deal here with models in which G is taken as horizontal gauge group G_H. Analysis of such models was first performed in ref. [1]. Two loop approach shows that electroweak parameters are significantly changed in such models in "proper" direction from the experimental point of view [1]. Even for the large class of non-supersymmetric models the electroweak mixing angle θ_W can be increased to $\sin^2\theta_W(M_Z) \sim 0.23$. Simultaneously the origin of various fermionic generations can be explained, and moreover, if one unifies a model into a single simple gauge group according to GUT scheme, the reasonable proton lifetime $\tau_p >\sim 10^{33}$ yrs is to be obtained.

To solve the problem completely we need to take into account on two loop level not only fermions associated with G_H but also intermediate gauge bosons generated by this group. An influence of these bosons was not discussed in details in former publications, where only preliminary results about horizontal gauge boson sector were given. The complete analysis of this problem is a task we undertake now.

In order to focus investigation on some class of models (G_H might in general be arbitrary) we will force G_H to satisfy few conditions. The detailed list of all requirements is given in [1]. We will quote the most important ones: $SU(3)_C \times SU(2)_L \times U(1)_Y \times G_H$ should be possible to be unified into simple gauge group G_{GUT} with complex representation ϕ_{GUT}; G_H is also assumed

to be simple group and unification of flavors within single generation should be permitted at some mass scale M_X before grand unification of flavors and generations occurs at scale $\mu_X > M_X$.

Corrections caused by loops involving horizontal bosons will be calculated from two loop solution of renormalization group equation for effective coupling constants α_μ $(\mu = 1,2,3,4)$ of $U(1)_Y,\ SU(2)_L,\ SU(3)_C$ and G_H respectively:

$$\alpha_\mu^{-1}(q^2) - \alpha_\mu^{-1}(Q^2) = (4\pi)^{-1}\beta_\mu \ln\frac{q^2}{Q^2} + (4\pi)^{-1}\sum_{\nu=1}^{4}\frac{\beta_{\mu\nu}}{\beta_\nu}\ln\frac{\alpha_\nu^{-1}(q^2)}{\alpha_\nu^{-1}(Q^2)} \tag{1}$$

where q^2, Q^2 are momenta and $\beta_\mu, \beta_{\mu\nu}$- first and second order β-functions. It is of particular interest to estimate mixed terms with $\beta_{i4} \equiv \beta_i^H (i = 1,2,3)$ responsible for two loop corrections involving horizontal bosons as well as terms with $\beta_4 \equiv \beta^H$. We will perform calculations for the class of theories satisfying the conditions quoted above. Such models were found in [1] and are listed once more in first columns of Table 1 with the following data: representation structure $\phi_{\rm GUT}$ in Young tableau language, the maximal number of fermionic generations n_G, and the complex part of fermionic representation after the breaking

$$G_{\rm GUT} \xrightarrow{\mu_X} G_f(\equiv SU(5)) \times G_H$$

(G_f assumed here to be $SU(5)$ is the unifying group of a single generation).

Using general formulas given in ref.[2] for the first and second order β-functions we arrive at

$$\beta^H = \frac{1}{3}\left\{\frac{11}{2}l(V^H) - \sum_k n_k l(\psi_k^H)\right\} \tag{2}$$

where V^H is the adjoint representation of G_H, $\psi^H \equiv \sum_k n_k \psi_k^H$ is decomposition of the complex part of ϕ_{GUT} into irreducible representations ψ_k^H of G_H, and $l(\phi)$ is Dynkin index [3] for representation ϕ.

The second order $\beta_{\mu\nu}$ functions for $\mu \neq \nu$ read

$$\beta_{\mu\nu} = -l(\psi^\mu)C_2(\psi^\nu) \tag{3}$$

where $C_2(\psi^\nu)$ is the Casimir eigenvalue for representation ψ^ν of the group G^ν. It can be expressed by the number of free parameters of G^ν $(dim\ \ G^\nu)$

and dimension of ϕ^ν ($dim\ \phi^\nu$):

$$C_2(\phi^\nu) = \frac{1}{2} l(\phi^\nu) \frac{dim\ G^\nu}{dim\ \phi^\nu} \tag{4}$$

By $\psi^{\mu(\nu)}$ in (3) we will understand nontrivial complex part of $\phi_{GUT} \longrightarrow (\psi^\mu, \psi^\nu)$ after decomposition $G_{GUT} \longrightarrow G^\mu \times G^\nu$ $(\mu, \nu = 1, 2, 3, 4)$.

Substitution of (4) to (3) gives

$$\beta_{\mu\nu} = -\frac{1}{2} \sum_{\psi \in \phi_{GUT}} l(\psi^\mu) l(\psi^\nu) \frac{dim\ G^\nu}{dim\ \psi^\nu} \tag{5}$$

The general form of the complex part of ϕ_{GUT} after decomposition $G_{GUT} \xrightarrow{\mu_X} G_f \times G_H$ will be

$$\phi_{GUT} \xrightarrow{\mu_X} \sum_k n_k(\mathbf{s}_k, \mathbf{h}_k) \tag{6}$$

where $\mathbf{s}_k$ are fermionic representations of G_f, ($G_f \xrightarrow{M_X} SU(3)_C \times SU(2)_L \times U(1)_Y$) and $\mathbf{h}_k$ are fermionic representations of G_H. Therefore we may write

$$\beta_i^H = -\frac{1}{2} dim\ G_H \sum_k \frac{n_k l(\mathbf{s}_k) l(\mathbf{h}_k)}{dim\ \mathbf{h}_k} \tag{7}$$

Let us note that the above formula does not depend on $i = 1, 2, 3$. It reflects the fact that unification within single generation (unification of flavours) occurs independently on the unification between distinct generations. Thus

$$\beta_3^H = \beta_2^H = \beta_1^H \tag{8}$$

and therefore two loop contributions from horizontal bosons are the same for all interactions. The values of β^H and β_i^H are also shown in Table 1.

Now we may proceed to evaluate corrections according to equation (1). It will be useful to keep in mind the schematic plot of effective couplings versus $t = \ln q^2$ as shown in Fig.1.

2 Corrections to the unifying mass M_X

Comparing $\mu = 3$ and $\mu = 2$ terms in (1) one finds that if horizontal boson coupling is switched off, i.e. $\beta_i^H \equiv 0$ then

$$\begin{aligned}
&\alpha_3^{-1}(M_H) - \alpha_2^{-1}(M_H) = (4\pi)^{-1}(\tilde{\beta}_3 - \tilde{\beta}_2)\ln\tfrac{M_H^2}{M_X^2} + \\
&+(4\pi)^{-1}\textstyle\sum_{j=1}^{3}\tfrac{\tilde{\beta}_{3j}-\tilde{\beta}_{2j}}{\tilde{\beta}_j}\ln\tfrac{\alpha_j^{-1}(M_H)}{\alpha_j^{-1}(M_X)}
\end{aligned} \tag{9}$$

where $\tilde{\beta}$ represents β-functions for $q^2 > M_H^2$ and $\alpha_3(M_X) = \alpha_2(M_X) \equiv \alpha_G(M_X)$ is assumed for α_G being the unified coupling constant of G_f

Similarly if one switches horizontal boson coupling on i.e. $\beta_i^H \neq 0$:

$$\begin{aligned}
\alpha_3^{-1}(M_H) - \alpha_2^{-1}(M_H) = \;& (4\pi)^{-1}(\tilde{\beta}_3 - \tilde{\beta}_2)\ln\tfrac{M_H^2}{\tilde{M}_X^2} + \\
& + (4\pi)^{-1}\textstyle\sum_{\nu=1}^{4}\tfrac{\tilde{\beta}_{3\nu}-\tilde{\beta}_{2\nu}}{\tilde{\beta}_\nu}\ln\tfrac{\alpha_\nu^{-1}(M_H)}{\alpha_\nu^{-1}(\tilde{M}_X)}
\end{aligned} \tag{10}$$

where $\tilde{M}_X$ is the unifying mass corrected by the presence of such coupling.

Subtracting (9) from (10) we arrive with

$$\ln\frac{\tilde{M}_X}{M_X} = \frac{1}{2\tilde{\beta}^H}\frac{\tilde{\beta}_3^H - \tilde{\beta}_2^H}{\tilde{\beta}_3 - \tilde{\beta}_2}\ln\frac{\alpha_H(M_X)}{\alpha_H(M_H)} \tag{11}$$

for $\alpha_H = \alpha_4$. Obviously $\tilde{\beta}^H = \beta^H$, $\tilde{\beta}_i^H = \beta_i^H$ and hence, because of (8), $\tilde{M}_X = M_X$. This result was obvious anyway; the corrections from β_i^H shift, according to (8), all plots $\alpha_i(q^2)$ in Fig.1 by the same amount leaving M_X at which intersection occurs - unchanged. We see that corrections from gauge boson sector of G_H do not change M_X up to two loop level. This means also that the expected proton lifetime $\tau_p \sim M_X^4$ should not be changed.

3 Corrections to the horizontal mass scale M_H

G_H is broken at M_H and this mass scale changes the behaviour of effective coupling constants with energy (see Fig.1). This fact is important in estimation of $\sin^2\theta_W(M_Z)$ expressed by $\alpha_2(M_Z)$ and $\alpha_1(M_Z)$ according to the

formula

$$\sin^2\theta_W(M_Z) = \left(1 + \frac{5}{3}\frac{\alpha_2(M_Z)}{\alpha_1(M_Z)}\right)^{-1} \tag{12}$$

Uncorrected M_H values calculated previously for various admissible models are quoted in the sixth column of Table 1. (for details of such calculation see [1]). Now we want to estimate corrections due to the presence of extra bosonic sector of G_H. Let us remark we have two regions of energy: one with $q^2 < M_H^2$ and the second with $q^2 > M_H^2$. All G_H bosons as well as those fermions of ϕ_{GUT} which transform with respect to real representations of G_f (or $SU(3)_C \times SU(2)_L \times U(1)_Y$) are assumed to decouple below M_H. Respective β-functions in these two regions will be denoted by β ($q^2 < M_H^2$) and $\tilde{\beta}$ ($q^2 > M_H^2$).

Subsequently we get from (1) for $\beta_i^H \equiv 0$:

$$\begin{aligned} 4\pi\Delta\alpha_3^{-1} &\stackrel{\text{def}}{=} 4\pi(\alpha_3^{-1}(M_Z) - \alpha_3^{-1}(M_X)) = \\ &= (\tilde{\beta}_3 - \beta_3)\ln M_H^2 + \beta_3 \ln M_Z^2 - \tilde{\beta}_3 \ln M_X^2 + \textstyle\sum_{j=1}^3 \frac{\beta_{3j}}{\beta_j}\ln\frac{\alpha_j^{-1}(M_Z)}{\alpha_j^{-1}(M_H)} \\ &\quad - \textstyle\sum_{j=1}^3 \frac{\tilde{\beta}_{3j}}{\tilde{\beta}_j}\ln\frac{\alpha_j^{-1}(M_X)}{\alpha_j^{-1}(M_H)} \end{aligned} \tag{13}$$

Both terms on the left-hand side of (13) are fixed; $\alpha_3(M_Z)$ is the experimental input while $\alpha_3(M_X) \equiv \alpha_G(M_X)$ is to be calculated to ensure perturbativity of a theory near M_X ([1]). Results for $\alpha_G(M_X)$ obtained numerically are also quoted for reference in Table 1.

Hence if horizontal boson sector is switched on one should get

$$\begin{aligned} 4\pi\Delta\alpha_3^{-1} &= 4\pi(\alpha_3^{-1}(M_Z) - \alpha_3^{-1}(\tilde{M}_X)) = \\ &= (\tilde{\beta}_3 - \beta_3)\ln \tilde{M}_H^2 + \beta_3 \ln M_Z^2 - \tilde{\beta}_3 \ln \tilde{M}_X^2 + \textstyle\sum_{j=1}^3 \frac{\beta_{3j}}{\beta_j}\ln\frac{\alpha_j^{-1}(M_Z)}{\alpha_j^{-1}(M_H)} \\ &\quad - \textstyle\sum_{j=1}^3 \frac{\tilde{\beta}_{3j}}{\tilde{\beta}_j}\ln\frac{\alpha_j^{-1}(\tilde{M}_X)}{\alpha_j^{-1}(\tilde{M}_H)} - \frac{\tilde{\beta}_3^H}{\tilde{\beta}^H}\ln\frac{\alpha_H^{-1}(\tilde{M}_X)}{\alpha_H^{-1}(\tilde{M}_H)} \end{aligned} \tag{14}$$

where $\tilde{M}_H$ is the corrected value of M_H.
Comparing (13) with (14) and keeping $M_X \sim \tilde{M}_X, \alpha_j(M_X) = \alpha_j(\tilde{M}_X) = \alpha_G(M_X)$ we obtain:

$$\frac{\tilde{M}_H}{M_H} = \left(\frac{\alpha_H(M_H)}{\alpha_H(M_X)}\right)^{\frac{\beta_3^H}{2\beta^H(\tilde{\beta}_3 - \beta_3)}} \tag{15}$$

To use the above formula we need to know the ratio $\frac{\alpha_H(M_H)}{\alpha_H(M_X)}$. This ratio will also be crucial in next section to determine the corrections to $\sin^2\theta_W$.
We receive on one loop level

$$\frac{\alpha_H(M_X)}{\alpha_H(M_H)} = 1 + (4\pi)^{-1}\beta^H \alpha_H(M_X) \ln\left(\frac{M_H}{M_X}\right)^2 \tag{16}$$

where $\alpha_H(M_X)$ should be estimated. Evolution of α_H and α_G for $M_X^2 < q^2 < \mu_X^2$ indicates (see Fig. 1) that

$$\alpha_H^{-1}(M_X) - \alpha_{GUT}^{-1}(\mu_X) = (4\pi)^{-1}\beta^H \ln\frac{M_X^2}{\mu_X^2} \tag{17}$$

$$\alpha_G^{-1}(M_X) - \alpha_{GUT}^{-1}(\mu_X) = (4\pi)^{-1}\beta_5 \ln\frac{M_X^2}{\mu_X^2} \tag{18}$$

where $\alpha_{GUT}(\mu_X)$ is the coupling constant of G_{GUT} at μ_X and β_5 is the one loop β-function for $G_f = SU(5)$ gauge theory.

Subtraction (17) from (18) yields to

$$\alpha_H(M_X) = [\alpha_G^{-1}(M_X) + (2\pi)^{-1}(\beta_5 - \beta^H)\ln\frac{\mu_X}{M_X}]^{-1} \tag{19}$$

All terms above are known except μ_X. To stay in perturbative region we need to satisfy $\alpha_{GUT}^{-1}(\mu_X) > 1$. Therefore

$$\frac{\mu_X}{M_X} \leq \kappa \tag{20}$$

where κ is estimated with the help of (18):

$$\kappa = \exp\{\frac{2\pi}{|\beta_5|}(\alpha_G^{-1} - 1)\} \tag{21}$$

(Note that $\beta_5 < 0$).
Values of κ estimated this way are also collected in Table 1. Let us remark that in all cases μ_X does not exceed the Planck mass $M_P \sim 10^{19} GeV$! Formulas (19)-(21) after substitution to (15)-(16) allow to calculate the relative correction $\tilde{M}_H/M_H$. We notice from (15), (16) and (19) it is maximal if

unification of flavors within generation occurs at the similar scale as the unification of generations i.e. $\mu_X = M_X$. In that case corrections $\delta M_H/M_H$, where $\delta M_H \stackrel{\text{def}}{=} \tilde{M}_H - M_H$ are tabularised (see Table 1). Their exact values depend strongly on the chosen model and vary from 2% to 12% of its initial value. Thus the presence of bosonic horizontal sector slightly increases the value of M_H.

4 Corrections to $\sin^2\theta_W$.

Finally we approach the most intriguing question which concerns corrections to $\sin^2\theta_W(M_Z)$. Formula (12) indicates that all changes in $s \equiv \sin^2\theta_W(M_Z)$ are due to changes in $\alpha_1(M_Z)$ and $\alpha_2(M_Z)$. These couplings can be calculated from the value $\alpha_G(M_X)$ using formula (1) and assuming the step-function behaviour of β-function near the M_H threshold. One easy finds this way:

$$\delta\alpha_1^{-1}(M_Z) \stackrel{\text{def}}{=} \alpha_1^{-1}(\beta_1^H \neq 0) - \alpha_1^{-1}(\beta_1^H = 0) = (4\pi)^{-1}\tfrac{\beta_1^H}{\beta^H}\ln\tfrac{\alpha_H^{-1}(M_H)}{\alpha_H^{-1}(M_X)} \qquad (22)$$

Due to (8) we have:

$$\delta\alpha_2^{-1}(M_Z) \stackrel{\text{def}}{=} \alpha_2^{-1}(\beta_2^H \neq 0) - \alpha_2^{-1}(\beta_2^H = 0) = \delta\alpha_1^{-1} \qquad (23)$$

Numerical estimation of $\delta\alpha^{-1} \equiv \delta\alpha_1^{-1} = \delta\alpha_2^{-1}$ is straightforward if one uses expressions (15) and (16) as the input.

Now we proceed to calculate δs - the correction to $\sin^2\theta_W$ - in terms of $\delta\alpha^{-1}$. Differentiating (12) with respect to $\alpha_1^{-1} = \frac{3}{5}(1-s)\alpha_0^{-1}$ and $\alpha_2^{-1} = s\alpha_0^{-1}$, where $\alpha_0 = 1/128$ is the electromagnetic coupling constant at M_Z we get

$$\delta s = \alpha_0\delta\alpha^{-1}\left(1 - \frac{8}{3}s\right) \qquad (24)$$

Maximal corrections correspond to $\mu_X = M_X$ and they are collected for all models at the end of Table 1. They are typically of order $\sim 10^{-3}$ and still increase $sin^2\theta_W$ up to tenth part of second loop corrections to $sin^2\theta_W$ caused by horizontal fermionic sector itself. Indeed as it was shown in ref.[1] fermionic sector in horizontal models is able on its own to increase $sin^2\theta_W$ from 0.216 to 0.233. These values are quoted for reference in Table 1. Let us notice that second loop horizontal bosons corrections, although small, can already

be larger than experimental accuracy in measurements of electroweak angle. Actually all LEP results and specific SLAC collider data give at present the world average value $sin^2\theta_W = 0.23156 \pm 0.00020$ [4]. Therefore these corrections may play soon a crucial role for acceptance or rejection various proposed models. It is already seen that $SU(2)_H$ is too small as horizontal gauge group candidate predicting $sin^2\theta_W$ slightly below the quoted value.
Let us finally comment on the possibility of massive neutrinos in presented models. All of them contain in its ϕ_{GUT} singlet part of G_f which makes space for righthanded neutrinos ν_R. Therefore mass terms for Dirac type ν's can be obtained. Table 1 shows possible representation space of ϕ_{GUT} in which ν_R's can be embedded.

Summarizing obtained results one concludes that extra gauge bosons related to additional gauge group G_H in theories beyond the SM do not remarkably change the evolution of gauge couplings of $SU(3)_C \times SU(2)_L \times U(1)_Y$ but positive features of these models will be even magnify if extra gauge bosons coupling is turned on.

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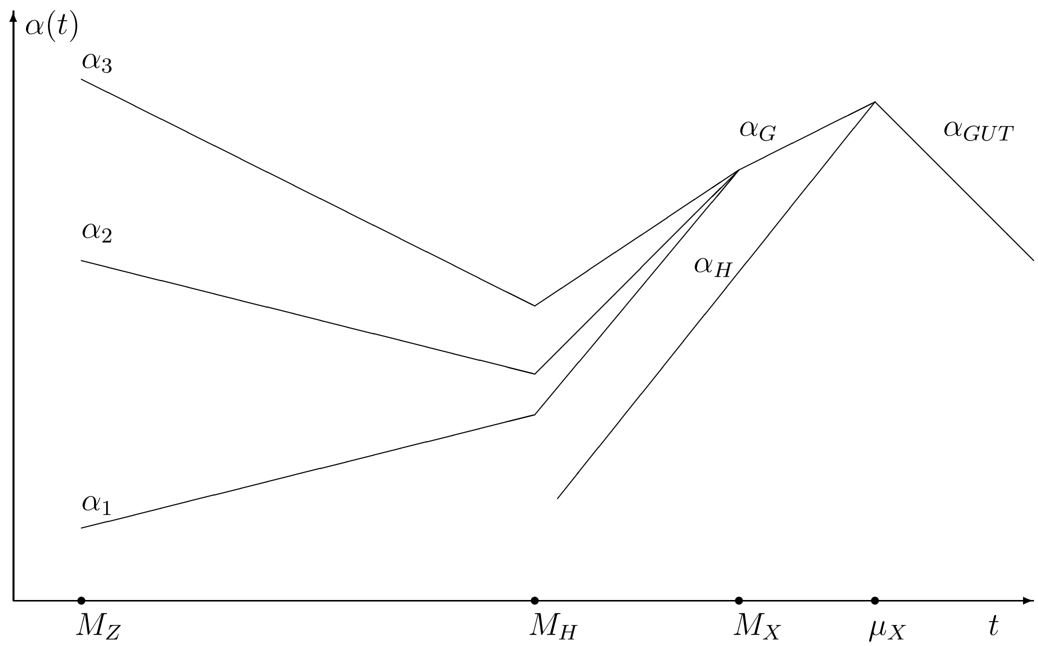

Figure 1: Schematic plot of running coupling constants versus energy $t = \ln q^2$ of strong, electroweak and horizontal interactions unified together at the mass scale μ_X in the framework of GUT.

G_{GUT}	G_H	ϕ_{GUT}	n_G	$G_f \times G_H$complex part	$M_H(GeV)$	$M_X(GeV)$	$\sin^2\theta_W$	α_G	β^H	β_i^H	$\kappa = \frac{\mu_X}{M_X}$	$\frac{\delta M_H}{M_H}(\%)$	$G_f \times G_H$ $\nu_R$space	$\delta \sin^2\theta_W$
$SU(7)$	$SU(2)$	$6\{\bar{1}\} + 2\{1^2\} + 6\{1^3\}$	4	$2(5^*,2) + 2(10,2) +$ $+ 4(10,2) + 8(10^*,1)$	4.4×10^9	1.21×10^{15}	0.228	0.3	-16	-15	133	7	$2(1,1) + 6(1,2)$	5×10^{-4}
$SU(8)$	$SU(3)$	$3\{\bar{1}\} + 3\{1^{\bar{2}}\} + 3\{1^3\}$	3	$(5^*,3^*) + (10,3)$ $+2(5^*,3^*) + 2(10,3)$ $+6(10^*,1) + 3(5^*,1)$ $+3(5,3^*)$	1.5×10^9	1.03×10^{15}	0.230	0.3	-9	-20	55	12	$3(1,1)$	1×10^{-3}
$SU(9)$	$SU(4)$	$19\{\bar{1}\} + 3\{1^2\} + \{1^3\} + \{1^{\bar{4}}\}$	4	$(5^*,4) + (10,4) +$ $+ 18(5^*,1) + (5,6) +$ $+ 3(5,4) + 20(1,4^*) +$ $+ 2(10,1) + (10,4^*) +$ $+ (10^*,6)$	1.3×10^{11}	1.32×10^{15}	0.233	0.3	-22	$-115/4$	3.8	7	$(1,1) + 3(1,6)$	7×10^{-4}
$SU(9)$	$SU(4)$	$13\{\bar{1}\} + 2\{1^3\} + \{1^{\bar{4}}\}$	4	$(5^*,4) + (10,4) +$ $+ 2(5,6) + 12(5^*,1) +$ $+ 15(1,4^*) + (10,4) +$ $+ (10,4^*) + (10^*,6) +$ $+ 2(10^*,1)$	1.3×10^{11}	1.25×10^{15}	0.234	0.3	$-46/3$	$-125/4$	3.8	9	$(1,1)$	1×10^{-3}
$SU(9)$	$Sp4$	$19\{\bar{1}\} + 3\{1^2\} + \{1^3\} + \{1^{\bar{4}}\}$	4	$(5^*,4) + (10,4) +$ $+ 17(5^*,1) + 3(5,4) +$ $+ (5,5) + (10,4) +$ $+ (10,1) + (10^*,5)$	5.0×10^{10}	1.93×10^{15}	0.233	0.4	$-74/3$	-41	3.1	12	$4(1,1) + 3(1,5)$	1.1×10^{-3}
$SU(9)$	$SO(5)$	$13\{\bar{1}\} + 2\{1^3\} + \{1^{\bar{4}}\}$	4	$(5^*,4) + (10,4)$ $+ 2(5,5) + 10(5^*,1)$ $+2(10,4) + (10^*,5)$ $+3(10^*,1)$	8.2×10^{10}	1.38×10^{15}	0.234	0.3	-28	-45	4.6	10	$(1,1) + 15(1,4)$	1×10^{-3}
$SU(10)$	$SO(5)$	$10\{\bar{1}\} + 3\{1^{\bar{2}}\} + 2\{1^3\}$	5	$(5^*,5) + (10,5) +$ $+ 2(5^*,5) + (10,5) +$ $+ 10(5^*,1) + 2(5,10) +$ $+ 5(10^*,1)$	6.6×10^{11}	1.17×10^{15}	0.231	0.3	$-194/3$	-48	3.5	5	$10(1,5) + 5(1,10)$	6×10^{-4}
$SU(11)$	$SO(6)$	$12\{\bar{1}\} + 4\{1^{\bar{2}}\} + 2\{1^3\}$	6	$(5^*,6) + (10,6) +$ $+ 3(5^*,6) + (10,6) +$ $+ 12(5^*,1) + 2(5,15) +$ $+ 6(10^*,1)$	3.3×10^{13}	1.37×10^{15}	0.234	0.1	$-116/3$	-33	13.6	2	$14(1,6) + 4(1,1)$	2×10^{-4}

Table 1: Examples of unified models with horizontal interactions, their parameters and second loop corrections from horizontal gauge boson coupling.